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Simulation of Small Chaos Generator to Study Chaotic Dynamics and Mathematical Network

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***Abstract*: This paper investigates the performance of a small scale chaos generator also referred to as a Chua's circuit, its simulation on Linear Technology Spice , Study of its waveform , its working and sustained conditions also explains the solving of the circuitry concepts by mathematical network theorem. The purpose of the paper is to show the chaotic dynamics of the circuit. The peripherals Synchronisation of the small scale chaotic generator as an application is also briefly explained by the authors in this paper.**

***Keywords*: Chua circuit, Linear Technology Spice, Chaotic dynamics, double scroll plot, Mathematical network, Cascaded drive response system.**

I. Introduction

Chua's circuit (also known as a Chua circuit) is a simple electronic circuit that exhibits classic chaos theory behaviour. It was introduced in 1983 by Leon O. Chua, who was a visitor at Waseda University in Japan at that time. Our purpose here is to change the respective values of the components and build another circuit with much more simplified values ,thus resulting in development of more simplified version of small chaotic generator [1] and studying its properties and dynamics with respect to original Chua's circuit.

II. Circuit Analysis and Simulation

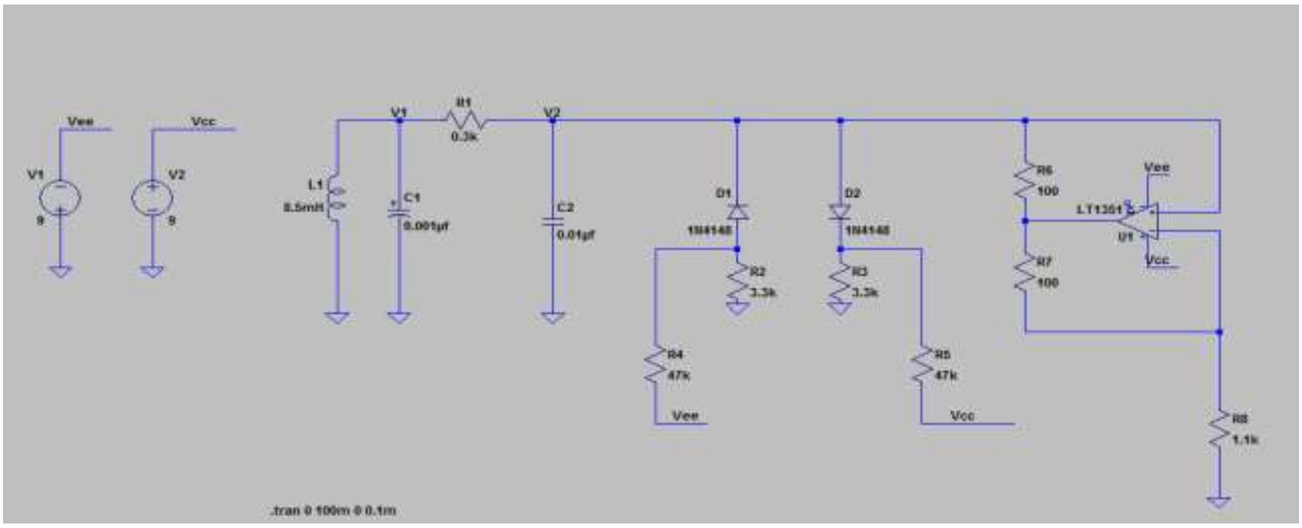

Figure 1(a): Chua circuit implemented on LT Spice

The above circuit is Chua's circuit it has been simulated on LT Spice, it is the circuit which was given by Prof .Leon Chua, the Values of components [2] is given as (L1=8.5mH, C1=0.001uF, C2=0.01uF ,R1=0.3Kohms ,R2=R3=3.3Kohms, R4=R5=47K, opamp 1351 (Simple 741 Opamp can be used practically), R6=R7=100ohms and R8=1.1Kohms. Vee =-9Volts and Vcc =+9 Volts) plotting the voltage (V1) measured across *C1* against the voltage(V2) measured across *C2*. The working of the circuit is as follows:

The inductor *L1* , the resistor *R1*, and the capacitors *C1* and *C2* make up a conventional linear oscillation circuit. With the rest of the circuit shorted out, this part would give damped oscillations. The op-amp and associate resistors have the effect of a **negative** resistance of size *–R8*. This is still a linear circuit element - it does not by itself give chaos, although it acts as the source of energy for the dynamics. The diode pair gives the nonlinearity in the circuit. The diodes simply serve to switch in the resistance *R2* in parallel with the resistance *–R8* when the voltage gets larger than the switch on voltage.

Necessary conditions for a set of evolution equations (assumed to be first order differential equations) to show chaos are:

- There must be at least three variables and equations
- There must be some nonlinearity

Kirchhoff's equations for the circuit, involving equations for the voltages V_1 across C_1, and V_2 across C_2, and the current I through the inductor *L1* satisfy these requirements

- The requirement of 3 equations is given by the inclusion of three reactive elements C_1, C_2, and *L*.
- The nonlinearity in the circuit comes form the piecewise linear effective negative resistance *f(V)* produced by the op-amp, diodes and associated resistors. Thus the *only nonlinearity in the effective circuit is the kink in the I-V characteristics f(V).*

2.1 Observations

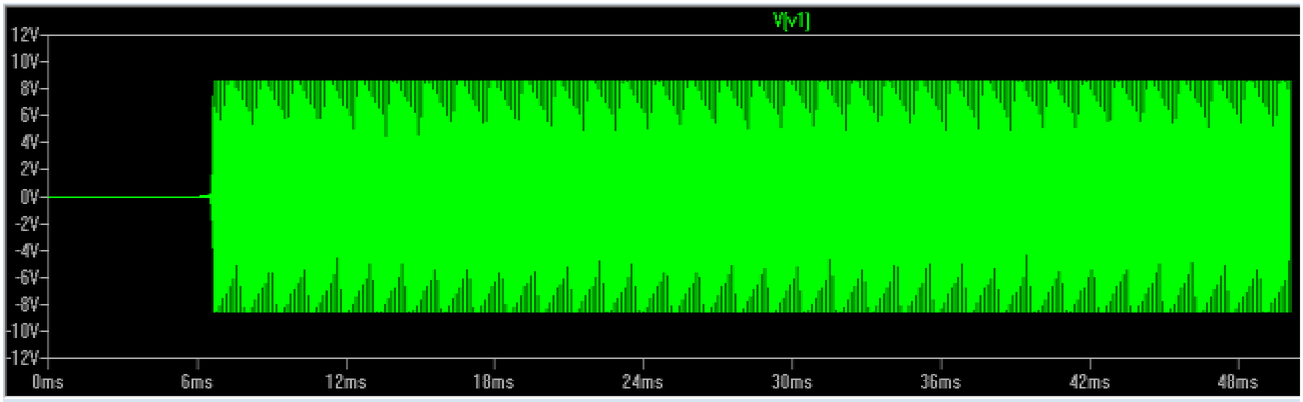

Figure 1(b): Simulation Result: Output waveform at voltage v1.

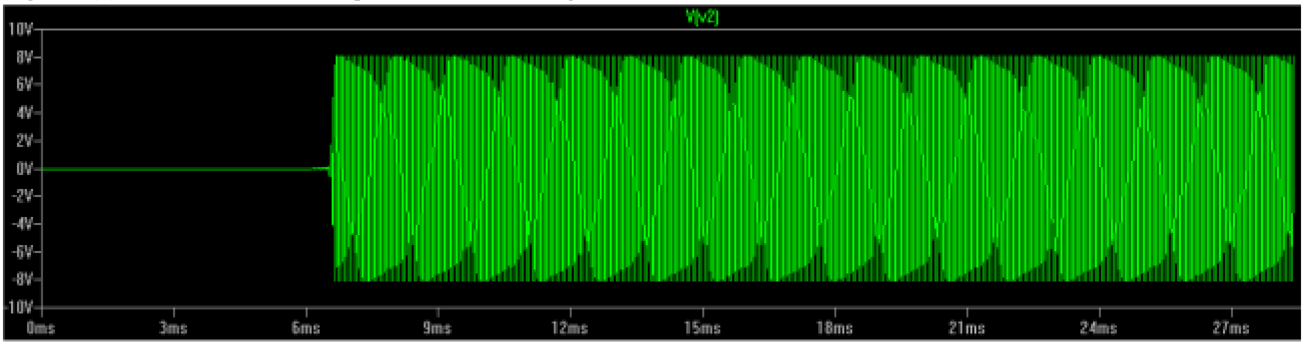

Figure 1(c): Simulation Result: Output waveform at voltage v2.

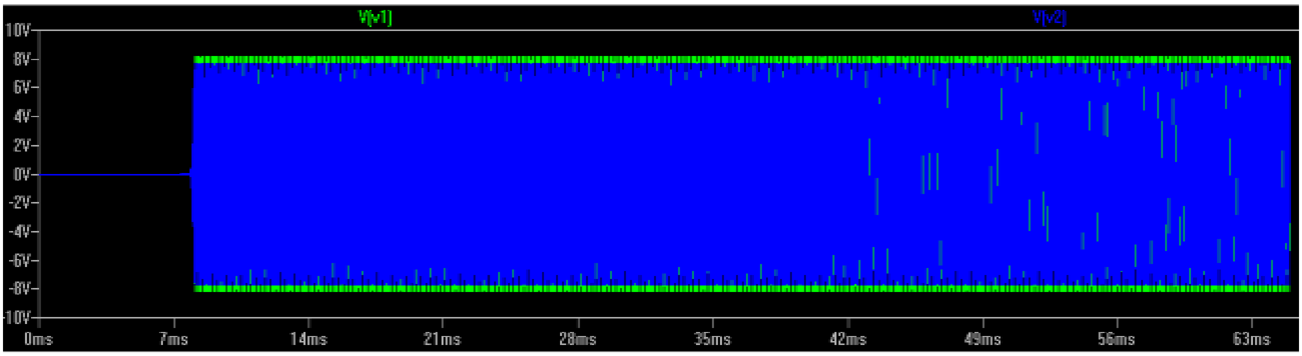

Figure 1(d): Simulation Result: Output waveform at voltage v1 and v2 together showing minimum chaos.

III. Modification of Circuit and its Analysis

To study the performance characterstics and the chaotic dynamics, we modified the values of the different components in the Chua's Circuit [3]; the Values modified are as follows:

- Value of R1 Component is changed to 1.6KOhms
- Value of Capacitance is changed to C1=47nF and C2=4.7nF
- Value of Resistors is also modified i.e. R6= R7=300 ohms and R8=1.1 Kohms

All other Values of the components remain same as it was in the previous circuit. On Circuit analysis and Simulation

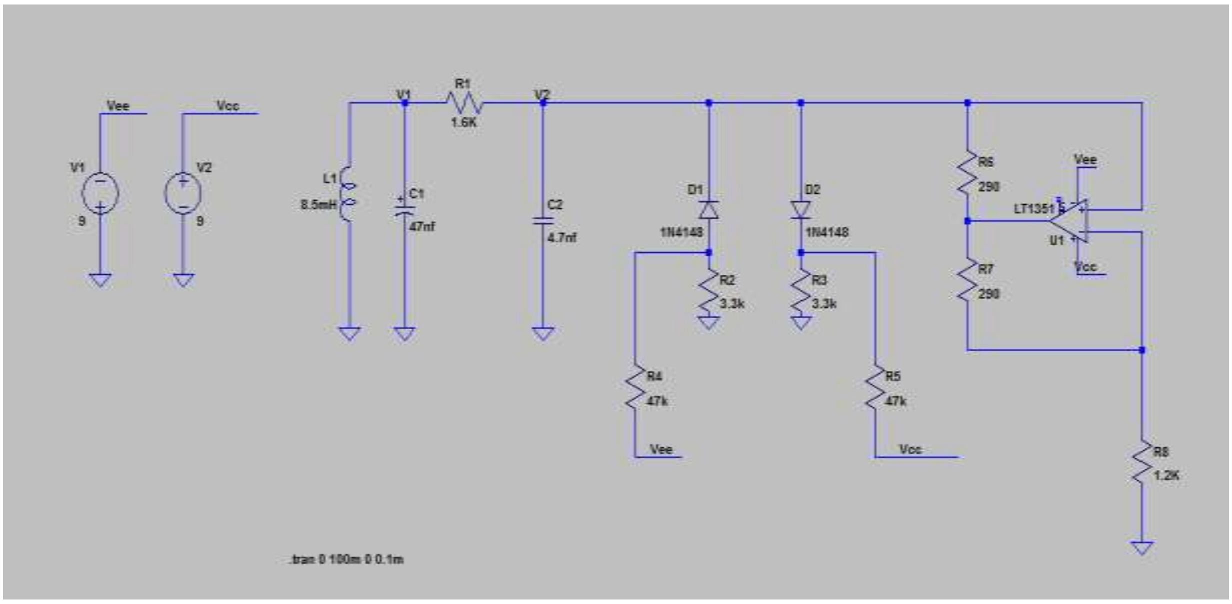

Figure 2(a): Chua circuit with variations in values of passive elements implemented on LT Spice

The above circuit is modified version Chua's circuit or a small chaos generator it also has been simulated on LT Spice and is compared with the previous pre defined circuit, the Values of components is given as (L1=8.5mH , C1=47nF , C2=4.7Nf ,R1=1.6Kohms ,R2=R3=3.3Kohms, R4=R5=47K, opamp 1351 (Simple 741 Opamp can be used practically), R6=R7=290ohms and R8=1.2Kohms. Vee =-9Volts and Vcc =+9 Volts) plotting the voltage (V1) measured across *C1* against the voltage(V2) measured across *C2*. This corresponds to the display on an *X-Y* Marging waves with probes connected across these capacitors. The initial values of the parameters used in the circuit , shows a simple periodic orbit (oscillation). The transition to chaotic dynamics can be found that decreasing *R1* or *C1*, (e.g. decrease *R1* in steps of

0.01 to 1.6K). The simulation compares well with what is actually seen on an oscilloscope. Chaos seems to develop via a sub harmonic cascade [4].

As an output the voltage V2 is almost 5 times to Oscillation of Voltage V1, indicating the generation of chaos or a high level amplification in the noise. This circuit proves to be much more efficient according to the performance waveform curves plotted.

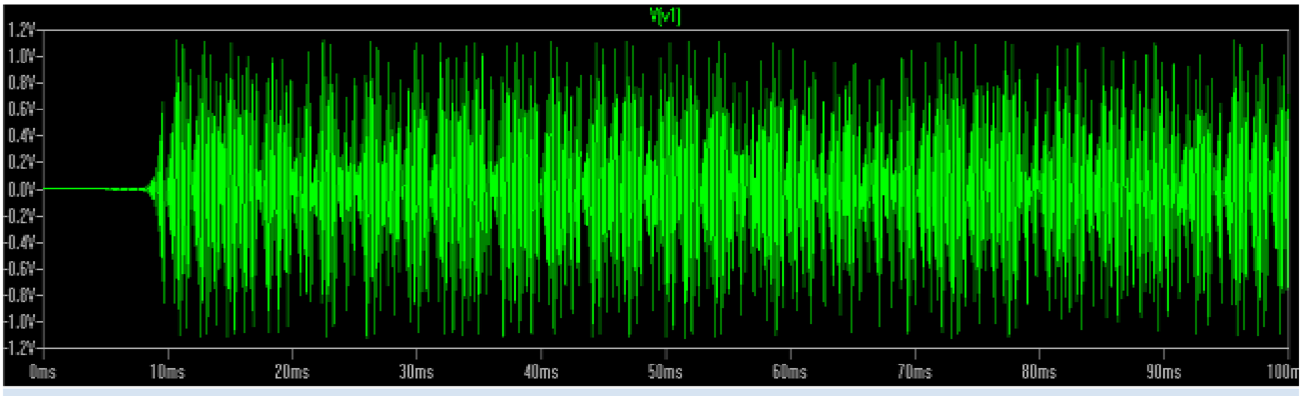

Figure 2(b): Simulation Result: Output waveform at voltage v1.

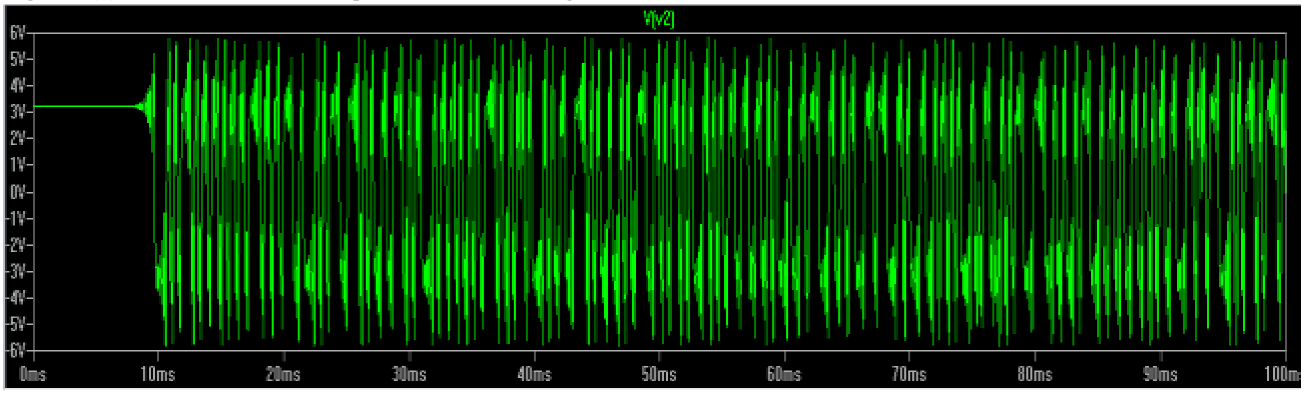

Figure 2(c): Simulation Result: Output waveform at voltage v2.

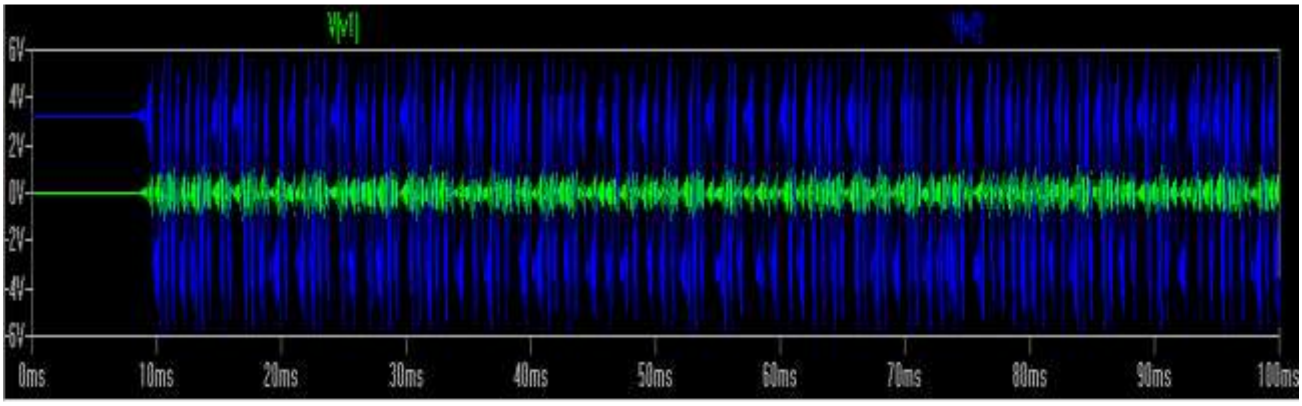

Figure 2(d): Simulation Result : Output waveform at voltage v1 and v2 together showing maximum chaos and chaotic dynamics can be compared with fig.1(d).

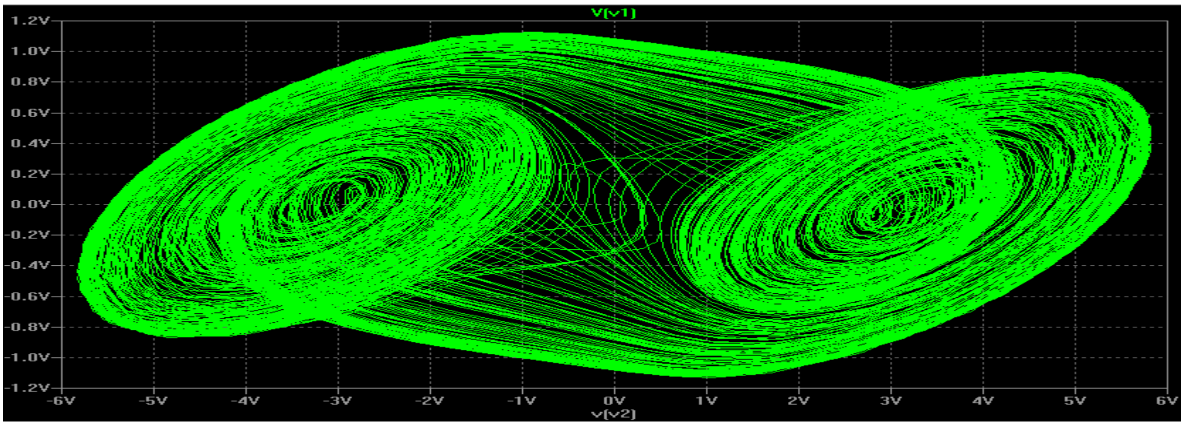

Figure 2(e): Simulation Result: if we change the x-axis to be v2 and the y-axis to v1, we get the beautiful double scroll plot seems like audible chaotic generation.

IV. Mathematical Modelling of Chua Network

Reducing Fig 2(a) to Fig 3(a) by neglecting and simplifying the network not involving operational amplifier thus , Fig 3(a) shows the reduced Chua's circuit that includes two capacitors, a resistor, an inductor and a nonlinear resistor NR or a diode we can say (a pair of negative resistors). Applying KCL and KVL [5], the Chua's circuit is described by three differential equations:

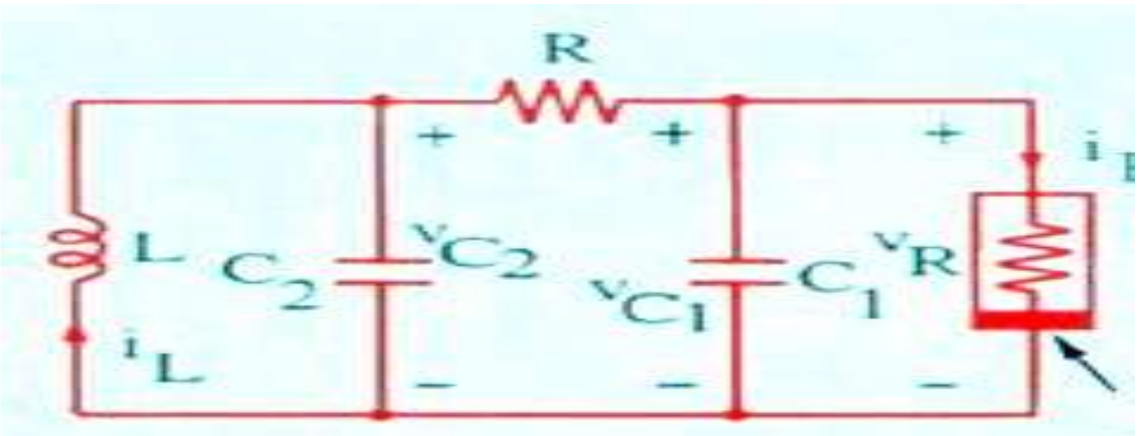

Figure 3(a): Reduced Chua circuit for mathematical network modelling

When we solve the above network by applying kcl and kvl three differential equations are obtained which are as follows

$C1\ \partial vC1/\partial t = (vC1-vC2)/R - f(vC1)$ -----(i)

$C2\ \partial vC1/\partial t = (v1-v2)/R + iL$ ------(ii)

$L\ \partial i1/\partial t = -vC2$ ------(iii)

. The Chua Equations are simpler than the Lorenz Equations in the sense that it contains only one scalar nonlinearity, whereas the Lorenz Equations contains 3 nonlinear terms, each consisting of a product of two variables (Pivka *et al*, 1996). In the original version studied in-depth in (Chua *et al*, 1986), $\phi(x)$ is defined as a *piecewise-linear* function

$\phi(x)=\triangle x+g(x)=m1x+12(m0-m1)[|x+1|-|x-1|]$

where m_0 and m_1 denote the slope of the inner and outer segments of the piecewise-linear function in Figure 1, respectively. Although simpler smooth scalar functions, such as polynomials, could be chosen for $\phi(x)$ without affecting the *qualitative* behaviors of the Chua Equations, a continuous (but not differentiable) piecewise-linear function was chosen strategically from the outset in (Chua *et al*, 1986) in order to devise a *rigorous* proof showing the experimentally and numerically derived *double scroll attractor is indeed chaotic*.

Thus this is how mathematical modelling of the chua circuit (Small scale Chaos Generator) is done .

V. Applications of Chua Circuit

One of the important applications of chaos is in secure or protected communication.One of the problems faced in the implementation of the communication system was that of synchronization. Efforts are in progress to perfectly match the chaotic systems at the transmitter and the receiver and that can only be done by possible peripheral synchronisation. With coupling and synchronizing identical chaotic systems methods, a message signal sent by a transmitter system can be reproduced at a receiver under the influence of a single chaotic signal through synchronization.

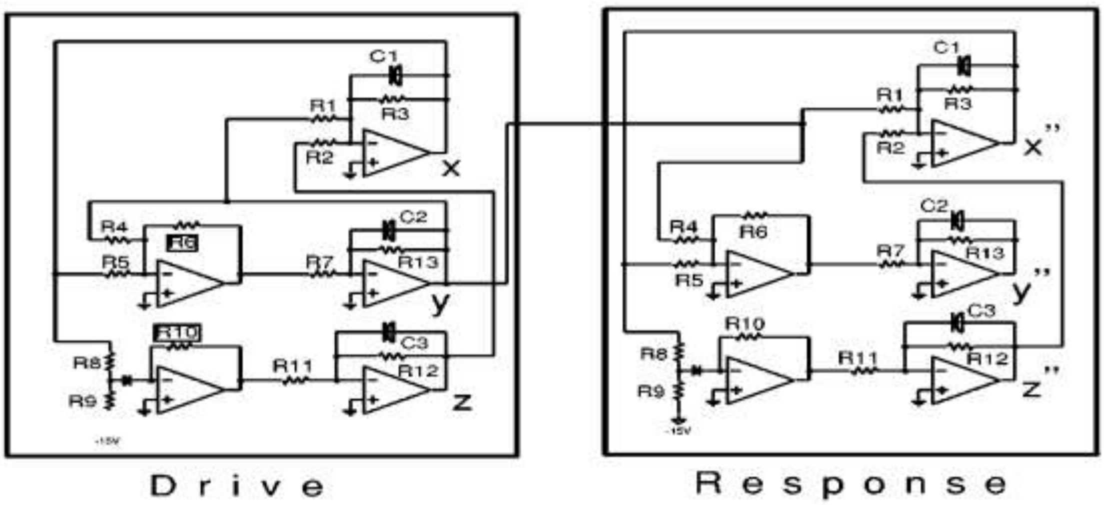

Figure 4(a) : A cascaded Drive Response System Piecewise linear Ro¨ssler circuits arranged for cascaded synchronization with R1=100 kohm, R2=200 kohm, R3=R13=2 Mohms, R4=75 kohm, R5=10 kohm, R6=10 kohm, R7=100 kohm, R8=10 kohm, R9=68 kohm, R10=150 kohm, R11=100 kohm, R12=100 kohm, C1=C2=C3=0.001 mF, and the diode is a type MV2101.

The above shown is the *cascaded* drive-response system[6] for the purpose of Synchronization of chaotic motions among coupled dynamical systems and it is an important generalization from the phenomenon of the synchronization of linear system, which is useful and indispensable in the field of communications. The idea of the methods is to reproduce all the signals at the receiver under the influence of a single chaotic signal from the driver. Therefore, chaos synchronization provides potential applications to communications and signal processing .Thus, Due to the fact that output signal can recover input signal, it indicates that it is possible to implement secure communication for a chaotic system.

VI. Conclusions

This paper studies the rich chaotic dynamics with variations in passive components in the chua's circuit through simulation on Linear Technology Spice and also focuses on mathematical network modelling of the circuit involving certain crucial mathematical equations .Finally application sustainability of chua's circuit through synchronisation technique of cascaded drive-response system is also illustrated in brief.

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Mr. Karan currently is a B.Tech third year student at Department of Electronics & Communication Engineering at the ITM University, Delhi (India). He has published three papers so far, two in national journal and one in International Journal. His research interests are in the areas of design of microprocessors-based systems, Spintronics, analysis of integrated circuits, antenna and wave propagation and data communication.